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Letter to the Editor

Suppression of chaos in a class of non-linear systems by disturbance observers

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1. Introduction

Perhaps all natural and physical systems are governed by non-linear laws of nature. The dynamics of most of such systems can be mathematically represented by non-linear differential or integral equations, which can be studied by analytical or numerical techniques. These techniques, in many instances, can successfully explain certain phenomena that are exclusive to non-linear systems. One such a phenomenon is chaotic behavior of systems. Chaotic behavior can be considered as both desirable and unwanted response of systems. For instance, chaotic systems can be used in secure communication systems to provide chaotic masking and modulation of transmitted messages; see, e.g., Refs. [1–5]. In most engineering systems, however, chaotic behavior is unwanted and should be suppressed. In the past decades, researchers have devised techniques to control chaotic behavior in non-linear systems; see, e.g., Refs. [1–3,6–17] and the references therein.

In this paper, it is shown that an effective means of suppressing the effects of non-linearities, and consequently possible chaotic behavior in a class of non-linear systems is the application of disturbance observers. Disturbance observers are useful tools that were originally proposed in Refs. [18,19] as means of estimating disturbances to linear systems and cancelling them subsequently. Later, the theory of disturbance observers was advanced in Ref. [20]. Presently, disturbance observers are successfully used in achieving robust stability and performance in motion control systems, for instance, robotic systems, high-speed machining systems, (micro) positioning systems, disk drives; see, e.g., Refs. [21–27] and the references therein. It appears that disturbance observers are mostly designed for linear systems. There are, however, some works where the application of disturbance observers to non-linear systems is reported; see Refs. [28–34]. The present paper illustrates that disturbance observers can make members of a certain class of non-linear systems behave linearly.

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The organization of the paper is as follows. In Section 2, the class of non-linear systems to be studied is presented. A non-linear system in this class has the property that its output is equal to the summation of the output of a stable single-input single-output (SISO) linear time-invariant system and a bounded disturbance. The bounded disturbance captures the effects of all non-linearities in the system. In Section 3, a disturbance observer is designed to estimate the effects of non-linearities in a system in the class under consideration (equivalently, the bounded disturbance) and cancel them subsequently. Having the non-linear effects cancelled, the system behaves linearly. In Section 4, an example is given to show that chaotic behavior due to a non-linearity in a Duffing-type system can be effectively suppressed by a disturbance observer.

2. Non-linear systems

In this section, a class of SISO non-linear systems is introduced. A member of this class, depicted in Fig. 1, is represented by

$$N: \quad \begin{cases} \dot{x}(t) = Ax(t) + f(x(t), t) + bu(t), \quad x(0) =: x_0, \\ y(t) = cx(t), \end{cases} \tag{1}$$

for all $t \geqslant 0$, where the state vector $x(t) \in \mathbb{R}^n$, the initial state vector $x_0 \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}$, the output $y(t) \in \mathbb{R}$, the coefficient matrix $A \in \mathbb{R}^{n \times n}$, the input (influence) vector $b \in \mathbb{R}^n$, the output (readout) vector $c = [c_{11}\ c_{12}\ \dots\ c_{1n}] \in \mathbb{R}^{1 \times n}$, and the non-linear function $f: \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is given by

$$f(x(t), t) = [f_1(x(t), t)\ f_2(x(t), t)\ \dots\ f_n(x(t), t)]^{\mathrm{T}}. \tag{2}$$

It is assumed that

(A1) The matrix A is Hurwitz.

(A2) The pairs (A, b) and (A, c) are, respectively, completely controllable and completely observable.

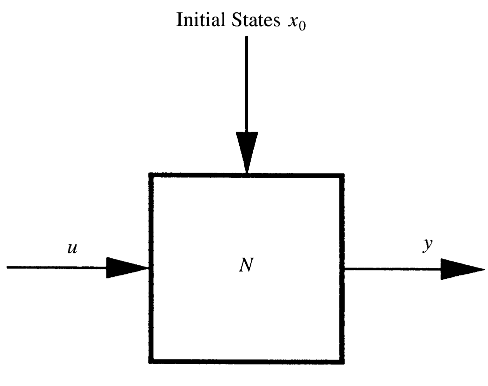

Fig. 1. The non-linear system N represented by Eq. (1).

(A3) The non-linear function f, though not exactly known, is norm bounded. More precisely,

$$\|f\|_\infty := \max_{1 \leqslant i \leqslant n} \sup_{x \in \mathbb{R}^n} \sup_{t \geqslant 0} |f_i(x,t)| \leqslant k_f < \infty, \tag{3}$$

where $k_f > 0$ is a constant real number.

Suppose that the system N exhibits a behavior exclusive to non-linear systems, such as chaotic behavior. Moreover, suppose that this behavior is deemed undesirable. Thus, the goal would be to suppress the non-linear behavior. This goal can be achieved by a disturbance observer as it will be shown later.

Before presenting the design of disturbance observers, some mathematical results are established. From Eqs. (1), it follows that the output of the system N can be written as

$$y(s) = H(s)u(s) + c(sI_n - A)^{-1}x_0 + d(s), \tag{4}$$

where $y(s)$, $u(s)$, and $d(s)$ are, respectively, the Laplace transforms of $y(\cdot)$, $u(\cdot)$, and the time function

$$d(t) = c\int_0^t \exp(A(t-\tau))f(x(\tau),\tau)\,\mathrm{d}\tau \in \mathbb{R}, \tag{5}$$

for all $t \geqslant 0$, I_n denotes the $n \times n$ identity matrix, and

$$H(s) = c(sI_n - A)^{-1}b. \tag{6}$$

The time function $t \mapsto d(t)$ has a useful property established as follows. Since by assumption (A1), the matrix A is Hurwitz, there exist constant real numbers $M > 0$ and $\sigma > 0$, such that

$$\|\exp(At)\|_\infty \leqslant M\exp(-\sigma t), \tag{7}$$

for all $t \geqslant 0$ (see, e.g., Ref. [35, p. 195]). Using inequalities (3) and (7) in Eq. (5), it is concluded that $t \mapsto d(t)$ is a bounded function of time. More precisely,

$$\|d\|_\infty := \sup_{t \geqslant 0} |d(t)| \leqslant \sum_{j=1}^{n} \|c_{1j}| M k_f / \sigma < \infty. \tag{8}$$

From Eqs. (4)–(6) and inequalities (7) and (8), it is concluded that the output of the non-linear system N is equal to the summation of the output of the stable SISO linear time-invariant system:

$$H: \quad \begin{cases} \dot{\bar{x}}(t) = A\bar{x}(t) + bu(t), \quad \bar{x}(0) =: \bar{x}_0 = x_0, \\ y_L(t) = c\bar{x}(t), \end{cases} \tag{9}$$

and the bounded function of time $d(t)$ for all $t \geqslant 0$, where the state vector $\bar{x}(t) \in \mathbb{R}^n$ and the output $y_L(t) \in \mathbb{R}$. By assumption (A2), the representation of the system H is minimal. The transfer function corresponding to H is irreducible and is that given in Eq. (6).

A conclusion to be drawn is that the non-linear system N can be equivalently represented by the linear system in Fig. 2. This system is denoted by H_{+d} and has a useful property to be exploited in the next section.

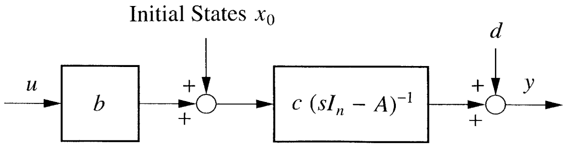

Fig. 2. The linear system H_{+d}. This system is an equivalent representation of the non-linear system N.

3. Linear behavior by disturbance observers

Representing the non-linear system N by the equivalent linear system H_{+d} in Fig. 2 is of great advantage, because the effects of non-linearities in N appear as the bounded disturbance $d(\cdot)$ in H_{+d}. Therefore, if one seeks to suppress the effects of non-linearities in N, then one should design a control law that suppresses the effect of $d(\cdot)$ in H_{+d}. The latter can be achieved by a disturbance observer that estimates $d(\cdot)$ and cancels it subsequently. Therefore, the goal of this section is to design a disturbance observer to make N behave linearly and, for instance, be free of chaotic behavior.

A disturbance observer added to the system H_{+d} is shown in Fig. 3. In this figure, $H_n(s)$ represents the nominal transfer function (mathematical model) corresponding to $H(s)$ in Eq. (6). In order to implement a disturbance observer, the filter $Q(s)$ is added to the system to make $Q(s)H_n^{-1}(s)$ a realizable (at least a proper) transfer function, because $H_n^{-1}(s)$ is often unrealizable. A successful design of a disturbance observer crucially depends on the design of $Q(s)$. Due to its important role, the design of $Q(s)$ has been extensively studied; see, e.g., Refs. [19,20,24,25]. It turns out that $Q(s)$ should be a low-pass filter of unity DC-gain. A typical form of $Q(s)$ is

$$Q(s) = \frac{\sum_{k=1}^{m-\rho} a_k(\tau s)^k + 1}{\sum_{k=1}^{m} a_k(\tau s)^k + 1}, \tag{10}$$

where ρ is at least equal to the relative degree of $H_n(s)$ and $a_k > 0$ and $\tau > 0$ are constant real numbers.

From Fig. 3, it is concluded that in the absence of the measurement noise ($w \equiv 0$),

$$\tilde{d}(s) = [H(s) - H_n(s)]v(s) + c(sI_n - A)^{-1}x_0 + d(s), \tag{11a}$$

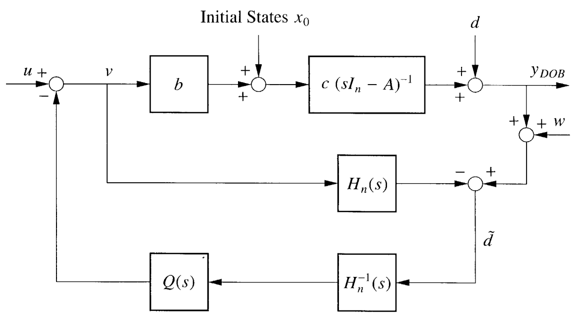

Fig. 3. A disturbance observer added to the system H_{+d} (equivalently N) to estimate d which captures the effects of non-linearities in N. An estimate of d is $\tilde{d}$ which is cancelled subsequently.

$$y_{DOB}(s) = [1 + H(s)(1 - Q(s))^{-1}Q(s)H_n^{-1}(s)]^{-1}H(s)(1 - Q(s))^{-1}u(s) + [1 + H(s)(1 - Q(s))^{-1}Q(s)H_n^{-1}(s)]^{-1}[c(sI_n - A)^{-1}x_0 + d(s)], \quad (11b)$$

where the output of the system is denoted by y_{DOB} to indicate a disturbance observer is implemented. Several comments regarding Eqs. (11) are made:

(1) The filter $Q(s)$ should be designed such that the transfer function

$$[1 + H(s)(1 - Q(s))^{-1}Q(s)H_n^{-1}(s)]^{-1} \quad (12)$$

is stable.

(2) By assumption (Al), the matrix A is Hurwitz. Thus, when Eq. (11a) is considered in the time domain, the effect of the initial state vector x_0 in this equation decays to zero. Moreover, $H(s) \approx H_n(s)$. Thus, from Eq. (11a), it is concluded that $\tilde{d}(\cdot)$ is an estimate of the bounded disturbance $d(\cdot)$.

(3) The filter $Q(s)$ is a low pass filter of unity DC-gain. Thus, from Eq. (11b), it follows that

$$y_{DOB}(s) \approx H_n(s)u(s). \quad (13)$$

That is, the effect of the bounded disturbance $d(\cdot)$ (as well as the decaying effect of the initial state vector x_0) in the system in Fig. 3 is suppressed, and the output of the system is approximately equal to that of the *linear* nominal system.

An implementation of the disturbance observer on the system N is shown in Fig. 4. The system in this figure is denoted by N_{DOB} to indicate a disturbance observer is added to N. The equivalence of N_{DOB} and the system in Fig. 3 asserts that the effects of non-linearities in N_{DOB} can be suppressed. That is, N_{DOB} would behave linearly.

Next, the performance of N_{DOB} is examined.

4. Example

In this section, an example is presented to illustrate the efficacy of disturbance observers in suppressing the effects of non-linearities and possible chaotic behavior in a non-linear system in the class of systems considered in this paper.

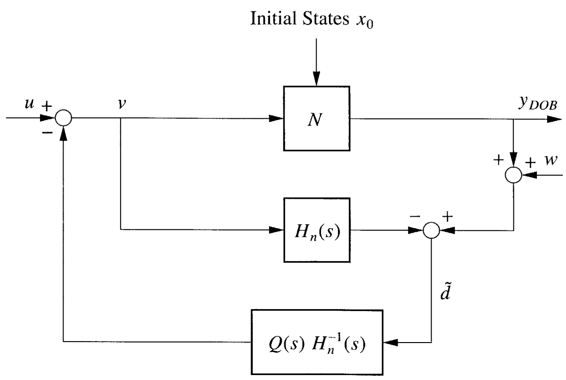

Fig. 4. The system N_{DOB}. This system is N to which a disturbance observer is added.

Consider a Duffing-type system represented by

$$\ddot{\xi}(t) + 0.1\dot{\xi}(t) + \xi(t) - \tan h(2\xi(t)) = 0.5\cos t, \quad \xi(0) = 0, \quad \dot{\xi}(0) = 0, \tag{14}$$

for all $t \geqslant 0$, where $\xi(t) \in \mathbb{R}$. Considering the first two terms in the expansion $\tan h(2\xi) = 2\xi - 8\xi^3/3 + \cdots$, system (14) should behave like a Duffing system (see, e.g., Refs. [36–38]). This fact is shown in the following.

By letting $x_1(t) = \xi(t)$ and $x_2(t) = \dot{\xi}(t)$ for all $t \geqslant 0$, system (14) can be written as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -0.1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \tan h(2x_1(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 0.5\cos t, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{15a}$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \tag{15b}$$

It is straightforward to verify that assumptions (A1)–(A3) hold for system (15). The output of system (15) is depicted in Fig. 5 and is denoted by y. Moreover, the trajectory corresponding to the solution of the system in the phase plane (x_1, x_2) is shown in Fig. 6. It is evident that system (15) exhibits a chaotic behavior typical to Duffing systems. The output of the linear system H, that is, system (15) in the absence of the non-linearity $\tan h(2x_1(\cdot))$, is depicted in Fig. 5 and is denoted by y_L. The steady state of y_L is the periodic function of time $t \mapsto 5\sin t$, which is obtained by applying results from the theory of linear oscillations.

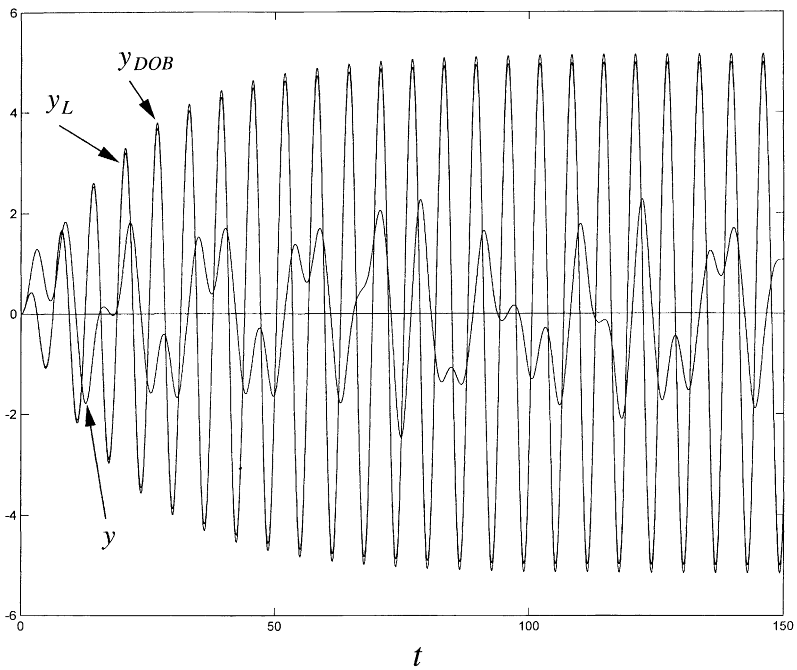

Fig. 5. Responses of the systems N, H (the non-linearity-free N), and N_{DOB}, denoted by y, y_L, and y_{DOB}, respectively, in the absence of the measurement noise w. It is evident that y is chaotic. Moreover, it is evident that y_{DOB} and y_L almost overlap. That is, the disturbance observer has suppressed the effect of the non-linearity.

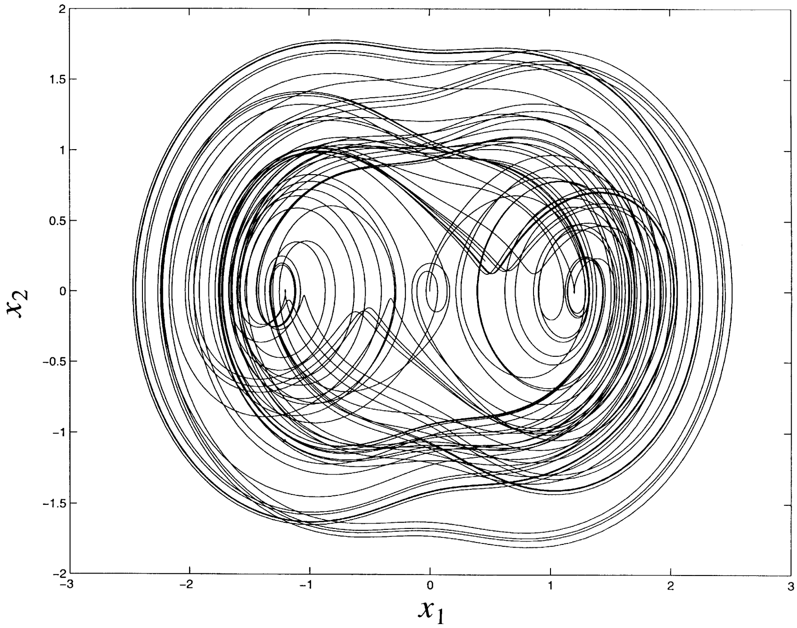

Fig. 6. Trajectory corresponding to the solution of the system N in the phase plane (x_1, x_2).

The difference between y and y_L is due to the non-linearity in system (15). It is now shown that the effect of this non-linearity, and consequently chaotic behavior, can be effectively suppressed by a disturbance observer. It is remarked that the control of chaotic behavior in Duffing systems is of great interest; see, e.g., Refs. [39–43].

The first step to the design of a disturbance observer is to obtain the transfer function $H(s)$ corresponding to the system H. This transfer function is readily determined from Eq. (6) and is given by

$$H(s) = \frac{1}{s^2 + 0.1s + 1}. \tag{16}$$

Having $H(s)$, a disturbance observer is implemented on system (15). The resulting system is N_{DOB} in Fig. 4, where $H_n(s) = H(s)$, the system N is that in Eq. (15), and

$$Q(s) = \frac{700}{s^2 + 9s + 700}. \tag{17}$$

The output of N_{DOB} in the absence of the measurement noise ($w \equiv 0$) is shown in Fig. 5 and is denoted by y_{DOB}. It is evident that y_{DOB} and y_L almost overlap, except that the former has a slightly larger amplitude. That is, the disturbance observer has successfully suppressed the effect of the non-linearity in system (15).

The effect of the measurement noise $w(\cdot)$ on the performance of the system N_{DOB} is studied next. Let $w(\cdot)$ be a band-limited white noise. The output of the system in the presence of $w(\cdot)$ is depicted in Fig. 7 and is denoted by y_{DOB}. This output is compared to that of the system H, denoted by y_L. It is evident that y_{DOB} and y_L almost overlap, except that the former has a slightly larger

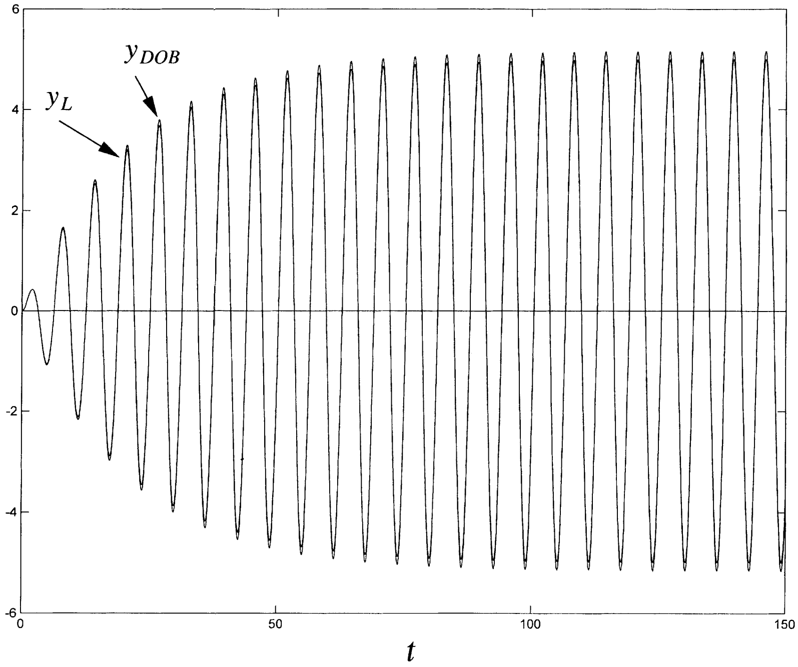

Fig. 7. Responses of the systems H and N_{DOB}, denoted by y_L and y_{DOB}, respectively, in the presence of the measurement noise w. It is evident that y_{DOB} and y_L almost overlap.

amplitude. That is, the disturbance observer is able to suppress the effect of the non-linearity in system (15) even in the presence of the measurement noise.

The magnitude of the control input $v(\cdot)$ in Fig. 4, which is applied to the system N, is assessed by plotting the time function $t \mapsto v(t)$. Plots of $t \mapsto v(t)$ in the absence and presence of the measurement noise $w(\cdot)$ are shown in Figs. 8(a) and (b), respectively. It is evident that the magnitude of $v(\cdot)$ is not large. For the purpose of comparison, the close-ups of the control inputs in the absence and presence of $w(\cdot)$ are plotted in Fig. 8(c) and denoted by $v_{w\equiv 0}$ and $v_{w\neq 0}$, respectively.

5. Conclusions

In this paper, the application of disturbance observers to suppress chaotic behavior in a class of single-input single-output non-linear systems was studied. A non-linear system in this class has the property that the effects of all non-linearities in the system can be captured in a bounded disturbance. Knowing this fact, it was shown how a disturbance observer can be designed to estimate the bounded disturbance (equivalently, the effects of non-linearities in the system) and cancel it subsequently. The disturbance observer is thus able to make the non-linear system behave linearly and, for instance, be free of chaotic behavior. The results of the paper were corroborated by using a disturbance observer to suppress chaotic behavior due to a non-linearity in a Duffing-type system.

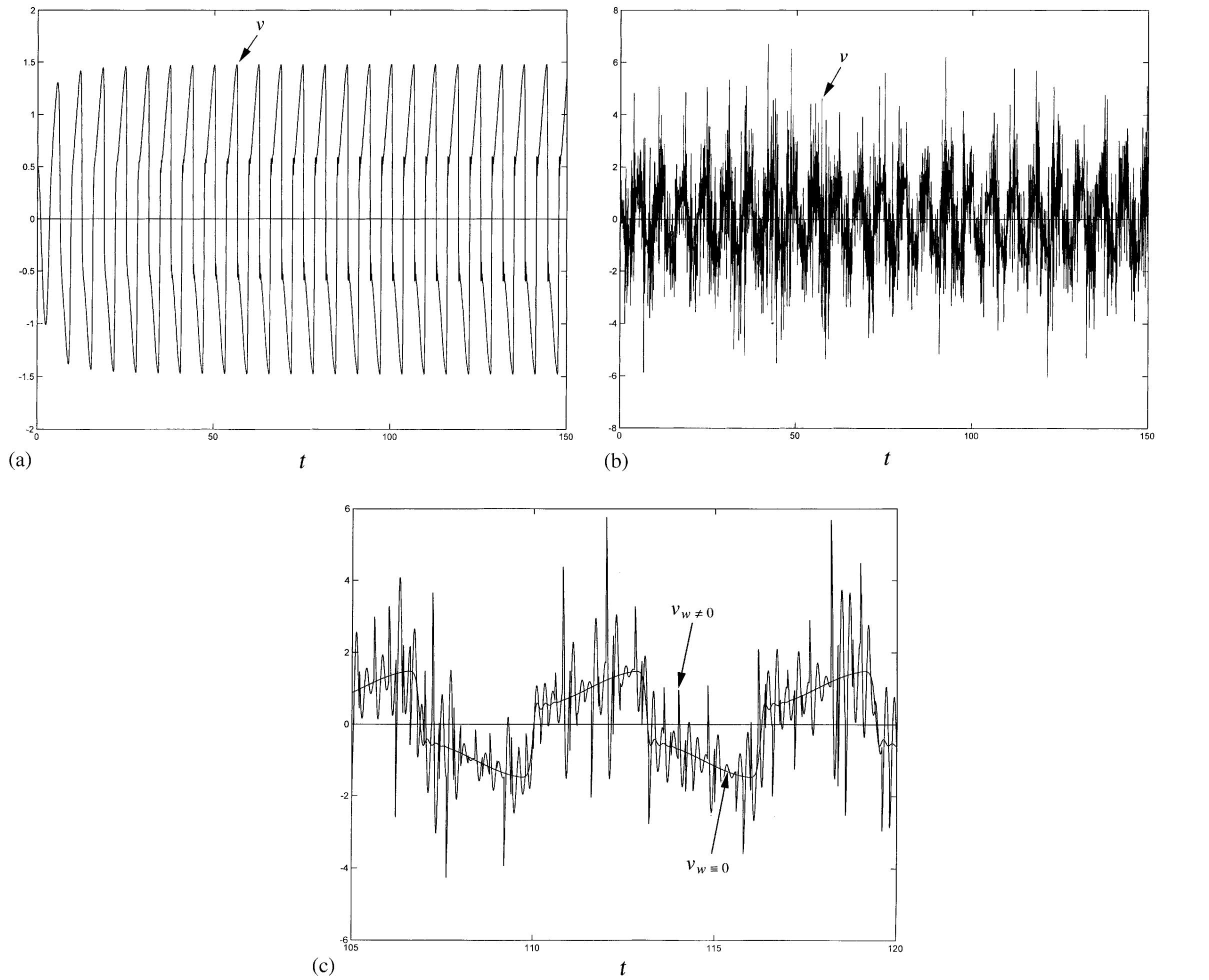

Fig. 8. (a) The control input v applied to the system N in the absence of the measurement noise w. (b) The control input applied to N in the presence of w. It is evident that the magnitude of v is not large. (c) Comparison of the control inputs in the absence and presence of w, denoted by $v_{w\equiv0}$ and $v_{w\neq0}$, respectively.

Three remarks are made regarding disturbance observers applied to the class of non-linear systems: (1) disturbance observers are linear systems, but yet they are able to suppress the effects of non-linearities in the systems; (2) disturbance observers can suppress the effects of non-linearities that are not exactly known; (3) the application of disturbance observers is not exclusive to chaotic systems. If non-linearities in a system cause an undesirable behavior, say limit cycle behavior, then disturbance observers can be used to suppress such a behavior. For instance, flutter in aircraft wings can be considered as limit cycle behavior. Thus, disturbance observers can be used to suppress flutter; work in this area is in progress and will be reported elsewhere.

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